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DURGAPUR INSTITUTE OF ADVANCED TECHNOLOGY & MANAGEMENT

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1st / 2nd Internal Test of 20 ____

BRANCH - ______________ SEMESTER - ______________

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MARKS OBTAINED

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Eg:- Prove a theorem "infer $(P \to Q) \wedge (Q \to R) \to (P \to R)$" also called chain rule.

Ans. The theorem to be proved.

Infer $(P \to Q) \wedge (Q \to R) \to (P \to R)$ — ①

premise $(P \to Q) \wedge (Q \to R)$ — ②

premise 1 $P \to Q$ — ③

Premise 2 $Q \to R$ — ④

Sub theorem. Infer $P \to R$ — ⑤

premise P — ⑥

By using ~~3~~ $[E\to, 3, 6]$ Q — ⑦

By using $[E\to, \cancel{7}\ 4, 7]$ R — ⑧

By using $[I\to, 6, 8]$ $P \to R$ — ⑨

By using $[I\to, 2, 9]$ $(P \to Q) \wedge (Q \to R) \to (P \to R)$

Conclusion.

— This is proof for chain rules

Eg:- Prove a theorem "Infer $(P \vee Q) \wedge (P \to Q) \to Q$".

— Called merging or eliminating duplicates.

Ans. The theorem to be proved,

$(p \vee q) \wedge (p \to q) \to q$ - - - ①

Premise $(p \vee q) \wedge (p \to q)$ - ②

Premise 2 By [E-∧, 2] $(p \vee q)$ - - ③

Premise 2 By [E-∧, 2] $p \to q$ - - ④

By [E-∨, 3, 4] q - - - ⑤

Hence By [I-→, 2, 5] $(p \vee q) \wedge (p \to q) \to q$. Conclusion

Proof by contradiction

Some times we make an false assumption related to be proof and proceed to a contradiction. And we said that what our assumption is wrong and thus theorem is proved — This process of theorem proving is called proof by contradiction

Eg: Prove a theorem infer $P \to P$ using contradiction rule.

Ans Theorem to be proved

infer $P \to P$ - - - ①

premise P - ②

Sub theorem $\sim P \to P \wedge \sim P$ - - ③

By [E-→, 3] premise $\sim P$ - - ④

By [2, 4, I-∧] - - $P \wedge \sim P$ - - ⑤.

By [E-∧, 4, 5] - P - - ⑥

∴ By [I-→, 2, 6] $P \to P$ - - - Conclusion

Hence $P \to P$ is proved.

Eg Prove a theorem Infer $P \to \sim\sim P$ using Contradiction Rule.

Ans The theorem to be proved is

$P \to \sim\sim P$ - - - ①

Premise P ---- (2)

Sub theorem $\sim P \rightarrow P \wedge \sim P$ ---- (3)

Premise $(\sim P)$ ---- (4)

By $[I-\wedge, 2, 4]$ $P \wedge \sim P$ ---- (5)

Hence $\sim P \rightarrow P \wedge \sim P$ is proved then ---- (5a)

By $[I-\sim, 2, 5a]$ $\sim(\sim P)$ ---- (6)

By $[I-\rightarrow, 2, 6]$ $P \rightarrow \sim(\sim P)$ ---- (7)

Hence It is proved that $P \rightarrow \sim(\sim P)$.

Eg:- Prove a theorem $(P \rightarrow Q) \wedge (\sim Q) \rightarrow (\sim P)$ using contradiction rule → also called Modus Tolence

Ans. The theorem to be proved

$(P \rightarrow Q) \wedge (\sim Q) \rightarrow (\sim P)$ ---- (1) (1a)

Premise $(P \rightarrow Q) \wedge (\sim Q)$ ---- By $[E-\wedge, 1]$ ---- (2)

premise 1 $(P \rightarrow Q)$ ---- By $[E-\wedge, 1]$ ---- (3)

Premise 2 $\sim Q$

Sub theorem to be proved ---- (4)

$P \rightarrow Q \wedge \sim Q$ ---- (5)

Premise P. By $[2, 5, E-\rightarrow]$ ---- (6)

Q ---- (7)

By $[I-\wedge, 3, 6]$ $Q \wedge \sim Q$ ---- (8)

By $[I-\sim, 5, 7]$ $\sim P$

~~By $[I-\rightarrow, 1a, 8]$ $(P \rightarrow Q) \wedge (\sim Q) \rightarrow \sim P$~~

Hence proved. $(P \rightarrow Q) \wedge (\sim Q) \rightarrow \sim P$.

(5) P.

Soundness and Completeness theorems

Theorem

Infer α is a theorem of natural deduction system if and only if α is valid.

Proof Assume that 'Infer α' is a theorem, then we have to proof that α is valid.

Since α is a theorem, then α is true for all interpretations and hence α is valid.

It is called soundness proof of theorem.

The converse is if α is valid then we have to proof that infer α is a theorem of Natural deduction system -- called completeness proof of theorem.

Now case 1:⇒ If α does not contain values T, F then Infer α contains only atoms and connection operators, and we can say α is a theorem because it is true ~~and~~ in all interpretation.

Case 2: ⇒ If α contains T and F then T can be replaced by formula $p \vee \sim p$ and F can be replaced by formula $p \wedge \sim p$, then we can obtain a equivalent β theorem in natural deduction system. Since $\alpha \cong \beta$, and α is valid then β is also valid.

Problem solving with natural Deduction System.

consider english text "If it is hot and humid, then it will rain. If it is humid then it is hot. It is humid now" - Prove that It will rain.

Let the following propositions.

P: It is hot

Q: It is humid

R: It will rain

Then the sentences can be represented in propositional calculus as follows.

"If it is hot and humid, then it will rain".

$\Rightarrow P \land Q \rightarrow R$.

"If it is humid then it is hot."

$\Rightarrow Q \rightarrow P$

and "It is humid now"

$\Rightarrow Q$.

therefore the given formulae are $P \land Q \rightarrow R,\ Q \rightarrow P,\ Q$.

And we have to proof

"It will rain"

$\Rightarrow R$.

$\therefore$ therefore the problem is

$$(P \land Q \rightarrow R) \land (Q \rightarrow P) \land Q \rightarrow R.$$

↳ we have to proof it

premise	$(P \land Q \rightarrow R) \land (Q \rightarrow P) \land Q$	①
premise 1	$(P \land Q) \rightarrow R$	②
premise 2	$Q \rightarrow P$	③
premise 3	Q	④
By [E $\rightarrow$, 3, 4]	P	⑤
By [I $\land$, 4, 5]	$P \land Q$	⑥
By [E $\rightarrow$, 2, 6]	R	⑦

Hence it is proved that

$$(P \wedge Q \to R) \wedge (Q \to P) \wedge Q \longrightarrow R.$$

∴ The conclusion is "It will rain" - proved.

Axiomatic System for propositional logic

Axiomatic approach is a direct method to prove theorems using given axioms, premises or hypotheses. It contains only three axioms and one rule for inference called modus ponence

Formal Axiomatic System for Propositional logic

This system generally has four components.

1) A countable set of symbols called alphabets, which is used to construct expressions in that system.
2) A set of well formed formulae which is a subset of all propositional expressions over the set of alphabets.
3) A set of axioms which is a subset of well formed formulae.
4) A set of Inference rule.

~~Here only two operators are used~~

~~(→, ~)~~. In axiomatic system the following things are included.

1. Parenthesis '(' & ')'
2. Operators '~' & '→'
3. atoms like P, Q, R, ---- etc.
4. Well formed formulae
5. Let α, β, γ are three Axioms.

Then three axioms which are included always true.