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G.T. ROAD, RAJBANDH, DURGAPUR - 12

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Problem with Truth table method

Truth table method is easy for evaluating consistency, inconsistency or validity of a formula. But the problem is that size of the table grows rapidly, Rather exponentially — which is uneconomical in the storage point of view. For this Natural Deduction System and Axiomatic System. It is based on few deductive inference rules.

Deductive Inference Rules

Inference is a process by which one formula is arrived at and affirmed on the basis of one or more other formulae accepted at starting point of the process.

Here E indicates elemination

P, P_K " atoms.

α_K, β, are formulae.

Deduction Rules:-

Rule 1: Introducing $\wedge$ (I-$\wedge$)

I-$\wedge$: If $P_1, P_2 \ldots P_n$ is proved then $P_1 \wedge P_2 \wedge \ldots P_n$ is also proved.

Rule 2: E-$\wedge$ (eliminating $\wedge$)

E-$\wedge$: If $P_1 \wedge P_2 \wedge \cdots P_n$ is proved then P_i is also proved.

Rule 3: Introducing $\vee$ (I-$\vee$)

I-$\vee$: If P_i is proved (any) then $P_1 \vee P_2 \vee \cdots \vee P_n$ is also proved.

Rule 4: Eliminating $\vee$ (E-$\vee$)

E-$\vee$: If $P_1 \vee P_2 \vee \cdots \vee P_n$ and $P_1 \rightarrow P$, $P_2 \rightarrow P$, $\cdots P_n \rightarrow P$ are given or proved then P is proved.

Rule 5: Introducing $\rightarrow$ (I-$\rightarrow$)

I-$\rightarrow$: If $\alpha_1 \rightarrow \beta$, $\alpha_2 \rightarrow \beta$, $\cdots$ $\alpha_n \rightarrow \beta$ i.e if $\alpha_i \rightarrow \beta$ or α_i infer β then $\alpha_1 \wedge \alpha_2 \cdots \wedge \alpha_n$ ~~ai~~ $\rightarrow \beta$ also infer β.

Rule 6: Eliminating $\rightarrow$ (E-$\rightarrow$).

E-$\rightarrow$: If $P_1 \rightarrow P$ and P_1 is given then Conclusion is P. rather if $P_1 \rightarrow P$, P_1 are given then P is proved.

This rule is called Modus Ponence

Rule 7: Introducing $\leftrightarrow$ (I-$\leftrightarrow$)

I-$\leftrightarrow$: If $P_1 \rightarrow P_2$ and $P_2 \rightarrow P_1$ are given then $P_1 \leftrightarrow P_2$ is proved.

Rule 8: Eliminating $\leftrightarrow$ (E-$\leftrightarrow$)

E-$\leftrightarrow$: If $P_1 \leftrightarrow P_2$ is given then $P_1 \rightarrow P_2$ and $P_2 \rightarrow P_1$ is proved.

Rule 9:- Introducing ~ (I-~)

I-~: If ~~P~~ $P \to P_1 \wedge \sim P_1$ is given ~~and also~~ ~~is given the result is $\sim P_1$, or $\sim P_1$ is proved~~ Then ~P is proved.

Rule 10:- Eliminating ~ (E-~)

E-~: if $\sim P \to P_1 \wedge$ ~~and~~ $\sim P_1$ is given then P is proved as a conclusion

Eg:-> Prove that $P \wedge (Q \vee R)$ follows from $P \wedge R$

Ans. So from the given problem it is given that

$\therefore P \wedge R \longrightarrow P \wedge (Q \vee R)$ — we have to proof.

let the premise is

$P \wedge R$ is proved ---- ①

$\therefore$ P is proved [From E-∧] -- ②

$\therefore$ R " " [From E-∧] -- ③

$\therefore (Q \vee R)$ " " [From I-∨] -- ④

$P \wedge (Q \vee R)$ " " [From 2, 4, and I-∧]

Therefore $P \wedge Q \longrightarrow P \wedge (Q \vee R)$ is true.

Dedection theorem:

If $\alpha_1 \wedge \alpha_2 \wedge --- \alpha_n \to \beta$ or $\alpha_1 \wedge \alpha_2 \cdots \alpha_n$ infer β, is a theorem of natural deductive system if and only if from $\alpha_1, \alpha_2, \cdots \alpha_n$ all infer β.

proof:- Here it is given that $\alpha_1 \to \beta$, $\alpha_2 \to \beta$. ---- $\alpha_n \to \beta$. then $\alpha_1 \wedge \alpha_2 \wedge --- \wedge \alpha_n \to \beta$

~~therefore by using Rule~~ is given in Rule 5 (I,→)

Now we have to proof that if $\alpha_1 \wedge \alpha_2 \wedge --- \wedge \alpha_n \to \beta$ holds, then $\alpha_1 \to \beta$, $\alpha_2 \to \beta$, --- $\alpha_n \to \beta$

let the premise be

$\cancel{\alpha_1 \wedge \alpha_2 \wedge \cdots \wedge \alpha_n \rightarrow \beta}$

$\cong \cancel{\sim(\alpha_1 \wedge \alpha_2 \wedge \cdots \wedge \alpha_n) \vee \beta}$

$\cong \cancel{\sim\alpha_1 \vee \sim\alpha_2 \vee \cdots \vee \sim\alpha_n \vee \beta}$

~~Therefore if $\sim\alpha_1 \vee \sim\alpha_2 \cdots \vee \sim\alpha_n \vee \beta$ is proved.~~

~~Then $\sim\alpha_1$ is proved~~

It is said that in the theorem

$\alpha_1, \alpha_2, \cdots \alpha_n$ infer β.

$\therefore \alpha_1 \rightarrow \beta, \alpha_2 \rightarrow \beta, \cdots, \alpha_n \rightarrow \beta.$

Now from Rule 5 (I, →) we have.

$\alpha_1 \wedge \alpha_2 \wedge \cdots \wedge \alpha_n \rightarrow \beta.$

Eg Proof following theorem

$(P \wedge Q) \wedge (P \rightarrow R) \rightarrow R.$

Ans. Left hand side

$(P \wedge Q) \wedge (P \rightarrow R)$. — — — ①

$(P \wedge Q)$ is proved by [E-∧] — ②

P is " " [E-∧] — ③

Q " " " [E-∧] — ④

$P \rightarrow R$ " " " [1, E-∧] — ⑤

R " " " [3, 5, E-→] — ⑥

$\therefore$ Therefore the conclusion is R.

$\therefore (P \wedge Q) \wedge (P \rightarrow R) \rightarrow R.$

Eg. Infer $(P \vee Q) \wedge (P \rightarrow S) \wedge (Q \rightarrow S) \rightarrow (S \vee P \vee Q)$

Ans From the premise

$(P \vee Q) \wedge (P \rightarrow S) \wedge (Q \rightarrow S)$ [The premise] - (1)

$(P \vee Q)$ is proved by (E-∧) -- (2)

$(P \rightarrow S)$ is proved by (E-∧) -- (3)

$(Q \rightarrow S)$ is proved by (E-∧) -- (4)

S is proved by [2, 3, 4, E-∨]

$(S \vee P \vee Q)$ is proved by [2, 5, I-∨]

Therefore it is proved that

$(P \vee Q) \wedge (P \rightarrow S) \wedge (Q \rightarrow S) \rightarrow (S \vee P \vee Q)$

Sub Proofs and scope rules:

While proving a theorem - sometimes we have to prove another theorem related ~~to~~ to the proof called sub theorem and concerned proof is called sub proof. It is done using proper scope rules.

For example

Prove a theorem: Infer $((Q \rightarrow P) \wedge (Q \rightarrow R) \rightarrow (Q \rightarrow P \wedge R))$

To prove this theorem we have to proof another theorem $Q \rightarrow P \wedge R$ related to this, concerned theorem is called sub theorem and proof is called subproof.

Ans. from the given problem we have the L.H.S. $(Q \rightarrow P) \wedge (Q \rightarrow R)$ . . . (0)

premise 1 $Q \rightarrow P$ . . . . (1)

premise 2 $Q \rightarrow R$ . . . (2)

Sub theorem $Q \rightarrow P \wedge R$ . . . (3)

premise Q . . . (4)

~~Prem~~ P By ~~premise~~ [1, 4, E→] --- (5)

R By [3, 4, E→] --- (6)

$P \land R$ By [5, 6, I-∧]

∴ $q \rightarrow P \land R$ Conclusion --- (7)

∴ $((q \rightarrow P) \land (q \rightarrow R)) \rightarrow (q \rightarrow P \land R)$ Conclusion. [using (a) and 7]

Eg:- Prove a theorem infer $P \land q \leftrightarrow q \land P$

Ans. The theorem to be proved is

infer $P \land q \leftrightarrow q \land P$

∴ the sub theorem infer $P \land q \rightarrow q \land P$ --- (1)

premise $P \land q$ --- (2)

By [E-∧, 2] P --- (3)

By [E-∧, 2] q --- (4)

By [I-∧, 4, 3] $q \land P$ --- (5)

~~By [I→, 2, 5] $P \land q \rightarrow q \land P$ Conclusion for (1)~~

By [I→, 2, 5] $P \land q \rightarrow q \land P$ --- (6)

Sub theorem $q \land P \rightarrow P \land q$ --- (7)

premise $q \land P$ --- (8)

By [E-∧, 8] q --- (9)

By [E-∧, 8] P --- (10)

By [I-∧, 10, 9] $P \land q$ --- (11)

By [I→, 8, 11] $q \land P \rightarrow P \land q$ --- (12)

From (6). [I-↔, 6, 12] $P \land q \leftrightarrow q \land P$ Conclusion.